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On a Distribution of Post-Partum Amenorrhea Period Following a Live Birth

THE post-partum amenorrhea (PPA) is the period of temporary sterility immediately following the termination of a pregnancy during which conception does not usually occur. It has been considered in several demographic, biological and family planning studies. It forms an important segment of the interval between two successive pregnancy terminations and the need to study its behaviour in the context of couple fertility cannot be over-emphasised, especially in India where nothing definite is known about the mean duration of PPA. Several studies reveal that in India the mean duration of this period is much longer than that obtained in Western studies *(e.g.* see Peckham, 1934 and Stix, 1940). However, Saxena (1966) found the mean duration of PPA fairly short as compared with its average length reported in other studies relating to different parts of India. It has been observed that the length of amenorrhea following a live birth is relatively constant for a woman whose child survives for more than a year. Factors associated with lactation are believed to be crucial in this variation. Tietze (1961), Potter (1963), Potter *et al. (1965),* Salber *et al.* (1966), Knodel and Walle (1967), Jain *et al.* (1970) and Saxena (1977) have tried to support, through direct or indirect evidence, that lactation affects resumption of menstruation after child birth. It may, however, be noted that the duration of lactation depends on variables like rural-urban residence and educational level of a woman.

On the basis of the original data on PPA collected in a demographic survey of Banaras, this paper examines the nature of observed distribution of PPA following first live-birth where the child had survived for more than a year. A mixture of two truncated Chi-square distributions has been proposed to describe the observed data on PPA and has been utilised to construct an attrition-table of the PPA following a live-birth.

The information collected in the Survey relates to the fertility history of a sample of 709 couples, covering nearly half the population depending on the clerical profession in the Banaras Hindu University. However, the information pertaining to PPA was available for only 425 women who had given at ieast one Uve-birth and were in the age range of 15-35 years at the time of survey. Out of the 425 first order live-births, 56 died in infancy and remaining 369 children survived more than a year (Table 1).

All the couples were personally interviewed. A woman was, however, contacted through her husband for collecting the data on the duration of PPA only when her husband could not furnish the same* Generally, the information on PPA could not be provided by male respondents at first contact. They were given a schedule (in local language) to be filled in consultation with their wives; they were requested to record the information only when the duration of PPA in their case was correctly known to them to the best of their knowledge. We might mention that *this sample is* from a traditional Hindu society *living in* the Eastern Uttar Pradesh, where traditional customs and taboos are *still* strictly observed. One of these taboos is that a woman during her menstrual-flow remains secluded and is not allowed to prepare food for other family members. The consciousness about PPA in this population is so great as to lend credence to the hope that resumption of menstruation after child birth, being an identifiable event, has been correctly reported by the couples.

The Model

The empirical distribution of PPA, as presented in Table I, is markedly skewed towards the right and the curve does not fall sharply after attaining a flattened peak. The observed distribution of PPA might have resulted from multiple patterns of breast-feeding among mothers. For simplicity, let us assume that there are two types of breast-feeding patterns : (i) of mothers who breast-feed their children for a longer duration as a result of which their mean PPA is comparatively longer; let a be the proportion of such mothers; and (ii) of mothers who discontinue breast-feeding quite early and consequently their average PPA may be *shorter o*ftheir proportion being I — «}.

Several distributions have been suggested for explaining the nature of the observed distribution of PPA; for example, in the studies of probability models, Talwar (1965) and Srinivasan (1966) assume its distribution to form an asymmetrical triangle, Singh (1963) treats the duration of PPA as constant for all live births, while Yadava (1966) takes it to be a chi-square distribution.

Assuming that the breast-feeding plays a major role in the resumption of menstruation after a child birth, the pattern of PPA may be suitably described by a mixture of two chi-square distributions by mixing proportions α and $(1 - \alpha)$ and with parameters n_1 and n_2 respectively. The choice of chi-square densities has been proposed by Yadava (1966) and it may be treated as a continuous analogue of triangular densities previously assumed by Talwar (1965) and Srinivasan (1966) for this purpose. Another consideration in selecting this density has been the simplicity in identifying the parameters n_1 and n_2 which are respective means of the two chi-square distributions.

From the experience of obstetrics, the minimum observed duration of PPA cannot be less than one month which implies that the probability that a woman will resume her menstruation within a month's time after delivery, is virtually zero. In order to incorporate this fact into the model we take the mixture of two truncated chi-square densities with truncation points on the left at one month's duration.

Let X denote the duration (in months) of PPA of a mother after a live birth, then the density function of X is given by,

$$f(x) = \alpha g(x; n_1) + (1 - \alpha)\, g(x; n_2), \tag{3.1}$$

where α is mixture proportion and

$$g(x; n_i) = [c(n_i)]^{-1} \frac{1}{2^{1/2 n_i}\,\Gamma(\frac{1}{2} n_i)} \exp\left(-\tfrac{1}{2}x\right) x^{1/2 n_i - 1} \; dx;$$

$$1 \leqslant x \leqslant \infty \tag{3.2}$$

$$c(n_i) = \frac{1}{2^{1/2 n_i}\,\Gamma(\frac{1}{2} n_i)} \int_1^{\infty} \exp\left(-\tfrac{1}{2}x\right) x^{1/2 n_i - 1}\, dx,$$

and $$n_i = n_1,\ n_2 > 0. \tag{3.3}$$

Application

The problem of constructing estimators for the parameters of mixture of distributions presents considerable difficulty. Moment estimates in the case of mixtures of two or more distributions are easier to obtain under certain simplified assumptions, but they do not provide satisfactory results. Efficient estimation procedures usually are found to yield intractable system of equations. For example, the likelihood function for a mixture of normals is so complicated that its maximisation, by iteration or any other technique, is a formidable task even on large high-speed computers (see Blischke, 1965). Rao (1940) has considerably simplified this problem by assuming equal variances of two normal distributions. Nevertheless, the calculations involved are still by no means easy and the constraint, so imposed, inhibits the application of mixture distributions in real situations. Problem of estimation becomes still more complicated when densities are truncated in the mixture.

The distribution given in (3.1) is a mixture of two truncated chi-squares. This has been fitted to the observed data of PPA, on "trial and error" basis, by utilising the *a priori* information concerning the parameters n_1 and n_2. As derived from the empirical distribution, the mean duration of PPA is about six months. This may be regarded as the weighted average of PPA of two groups of women, the mean PPA being greater than six months for one group and less than that for other group. Keeping this in view, we start with the pair of values taken from all the combinations of the trial values of n_t and n_2 (n_1 = 10(1) 13 months and n_3 = 3(1) 5 months) and estimated the mixing proportion a analytically by equating the mean of the observed PPA with the theoretical expression for the mean obtained from the distribution given in (3.1). Tables of incomplete Gamma function by Pearson (1957) have been used for solving the equation for a and also for obtaining theoretical frequencies corresponding to each pair n_1 n_3 (n_1 = 10, 11,12 and 13 months and n_2 = 3, 4 and 5 months). It is found that for n_l = 12 months, n_3 = 4 months (trial values) and $\hat{n}$ = 0.282, the distribution given in (3.1) provides a good fit to the observed distribution of PPA. Expected frequencies computed on the basis of these estimates of the parameters n_x, n_a and a along with the observed distribution of PPA have been given in Table 1.

An Attrition-Table for Post-Partum Amenorrhea

Since the fitted distribution gives a close fit, the same mixture distribution of two truncated chi-squares may reasonably be employed to graduate the frequency of mothers having the termination of PPA period between x to x + 2 months of delivery to be denoted by $2{*}x$ starting from a hypothetical cohort of 1,000 mothers from the point when they complete one month after the delivery.

Given these $2^{d}x$ values the other columns of the attrition-table are constructed in the same manner as a life table. The attrition-table is applicable as soon as a mother completes her first amenorrhoeic month from the delivery.

Conclusions

Distribution of many characteristics, particularly quantitative ones, may show variations markedly with certain unobservable variate. In the present context, the distribution of PPA may vary with the breast-feeding pattern which, in many situations, may be difficult to ascertain in a sample of mothers and may be treated as unobservable variate depending upon various factors *viz.*, rural urban residence, educational level, dietary habits and health condition of mothers. Mixture distributions are well suited in such situations and may be employed for separating a heterogeneous population into more homogeneous sub-populations with respect to the unobservable variate.

A mixture of two truncated chi-square distributions has successfully described the observed data on the duration of PPA. This mixed model has separated the given population of mothers into two homogeneous sub-groups—one group consists of those mothers (about 28 per cent) for which the PPA is longer and its mean duration is about 12 months, and the other group comprises all those women (about 72 per cent) with shorter PPA's, with a mean duration of about 4 months. The weighted mean, however, depends upon the mixing proportion of mothers in the two sub-groups. This could be one of the reasons for the large variation in the mean duration of PPA reported in different studies conducted in India.

Generally, the data on the timing of certain events, particularly the demographic ones, are found distorted. This deficiency in the data may be due to digit preference; inadequate sample size; and heterogeneity of population resulting from altogether different modes of behaviour of some segment of the population in which the events are delayed. In such situations, however, the unimodel distributions are found less suitable to graduate the whole series of the observed data. And thus, in many practical applications, the choice of the mixture of suitable distributions for graduating the observed data may be more appropriate. The present paper provides one such example.

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Appendix

TABLE 1—OBSERVED AND EXPECTED DISTRIBUTION OF POST-PARTUM AMENORRHEA FOLLOWING FIRST LIVE BIRTH

Duration of Post-Partum Amenorrhea in months	*Number of Mothers*	
	Observed	*Expected* $\hat{n}_1 = 12$ *months,* $\hat{n}_2 = 4$ *months and* $\hat{\alpha} = 0.282$
1-3	98	101.4
3-5	90	83.5
5-7	60	58.5
7-9	38	37.9
9-11	27	28.3
11-13	25	21.2
13-15	14	15.3
15-17	8	10.5
17-19	5 } 9	6.7 } 12.4
19-21	2	4.2
21+	2	1.5
Total	369	369.0

$\chi^2 = 3.05$

$\chi^2_{0.05[5]} = 11.07$

TABLE 2—AN ATTRITION-TABLE OF POST-PARTUM AMENORRHEA PERIOD WITH THE ORIGIN AT THE FIRST MONTH

x	1_x	2^dx	2^qx	2^Lx	T_x	e_x^0
1	1000	275	0.27500	1725	5362	5.3620
3	725	226	0.31172	1224	3637	5.0165
5	499	159	0.31864	839	2413	4.8357
7	340	103	0.30293	577	1574	4.6294
9	237	77	0.32489	397	997	4.2067
11	160	57	0.35625	263	600	3.7500
13	103	41	0.39800	165	337	3.2718
15	62	28	0.45161	96	172	2.7741
17	34	18	0.52941	50	76	2.2352
19	16	11	0.68750	21	26	1.6250
21	5	5	1.00000	5	5	1.0000
23	0	0	—	—	0	0

Abbreviations :

x = Period of PPA in months.
1_x = Number of mothers in exactly x-th month of PPA.
2^dx = Number of mothers who resume menstruation between x to $x + 2$ months of PPA.
2^qx = Probability of resuming menstruation between x to $x + 2$ months.
2^Lx = Mother-months lived by cohort between x to $x + 2$ without resumption of menstruation.
T_x = $\sum_x 2^Lx$.
e_x^0 = Expected number of months for the completion of PPA period at the x-th month of PPA.